NASA TECHNICAL TRANSLATION NASA TT F-11,691

NASA TT F-11,691

APPLICATION OF THE METHOD OF CHARACTERISTICS TO CALCULATIONS OF NONEQUILIBRIUM SUPERSONIC AIR FLOW PAST BLUNTED CONES

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Translation of "Primeneniye Metoda Kharakteristikh k Raschetu Neravnovesnogo Obtekaniya Zatuplennykh Konusov Sverkhzvukovym Potokom Vozdukha"
In: Vychislitel'nyye Metody i Programmirovaniye.
Chislennyye Metody v Gazovoy Dinamike
Sbornik Rabot Vychislitel'nogo Tsentra MGU,
No. 7, Moscow University Press, pp. 197-205, 1967

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, DC 20546 MAY 1968

Symbols

$p \cdot p_\infty$	pressure;
$T \cdot T_\infty$	temperature;
$\rho \cdot \rho_\infty$	density;
$w \cdot \sqrt{\frac{p_\infty}{\rho_\infty}}$	velocity of flow;
$v_x \sqrt{\frac{p_\infty}{\rho_\infty}}$; $v_y \sqrt{\frac{p_\infty}{\rho_\infty}}$	projection of velocity on coordinate axis;
θ	angle of inclination of velocity to x axis;
M	Mach number;
α_k	mass concentration of k-th component (N_2, O_2, NO, O, N, N_2^+, O_2^+, NO^+)
$\mu_k \cdot \mu_\infty$	molecular weight of k-th component;
$\mu \cdot \mu_\infty$	molecular weight of mixture;
$h_k \cdot \frac{p_\infty}{\rho_\infty}$	enthalpy of k-th component;
$e_{vi} \frac{p_\infty \mu_\infty}{\rho_\infty}$	oscillatory energy of component. ($i = N_2, O_2$).

The subscript "∞" relates to the parameters of the oncoming flow.

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ABSTRACT. This article deals with the application of the method of characteristics to calculations of the supersonic region in the flow near a blunt body, and is based on the available data obtained from detailed analyses of dissociation and ionization processes occurring beyond a shock wave. The boundary conditions on the body and the shock wave are used in the form which coincides with the case of a perfect gas with constant heat capacity, because all physical and chemical processes are considered frozen when passing through a shock front. Calculations of the flow field between the body and shock wave were carried out progressively by regions bounded by the reflected characteristics. The results of calculations of the air flow past blunt cones with 1.5 cm bluntness radius at $T_\infty = 250°$, $p_\infty = 10^{-3}$ atm., $M_\infty = 13$ are presented as an illustrative example and are compared with those for an equilibrium flow with $\kappa = 1.4$. The comparison of pressure and density distributions along blunt cones with apex angles of 10°, 0°, -10° and -20° for ideal, equilibrium and nonequilibrium flows is presented in graphical form.

As a blunt body flies with high supersonic velocity, the presence of nonequilibrium physical-chemical conversions in the air may have a considerable influence on the flow properties [1-3]. /197[1]

In [1], a systematic investigation of nonequilibrium flow about a sphere over a broad range of Mach numbers, oncoming flow pressures and body dimensions is presented; the subsonic and transonic flow areas are calculated right up to a certain ray AB (Figure 1) lying fully in the supersonic flow area. This work is dedicated to the usage of the characteristics method to calculate the supersonic flow area near a blunt body.

The movement of the reacting mixture of ideal gases, ignoring viscosity, heat conductivity and diffusion, is described by the following equations:

$$\frac{\partial v_x}{\partial x} v_x + \frac{\partial v_x}{\partial y} v_y + \frac{1}{\rho} \cdot \frac{\partial p}{\partial x} = 0,$$

$$\frac{\partial v_y}{\partial x} v_x + \frac{\partial v_y}{\partial y} v_y + \frac{1}{\rho} \cdot \frac{\partial p}{\partial y} = 0,$$

[1] Numbers in the margin indicate pagination in the foreign text.

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{v_x}{\rho} \cdot \frac{\partial \rho}{\partial x} + \frac{v_y}{\rho} \cdot \frac{\partial \rho}{\partial y} + j\frac{v_y}{y} = 0,$$

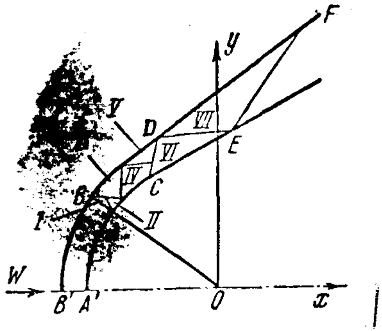

Figure 1. Diagram of Flow Around a Blunt Cone

$$h + \frac{w^2}{2} = h_0; \quad \left(h = \sum_{k=1}^{n} h_k \alpha_k\right),$$ /198

$$p = \frac{\rho R T}{\mu}; \quad \left(\frac{1}{\mu} = \sum_{k=1}^{n} \frac{\alpha_k}{\mu_k}\right).$$

J = 0 for a plane flow,
1 for an axisymmetrical flow.

In order to determine the composition and thermodynamic properties of the gas, a system of equations of continuity for the components of the gas is added in the form

$$\frac{\partial \alpha_k}{\partial x} v_x + \frac{\partial \alpha_k}{\partial y} v_y = \frac{1}{\rho}\left(\frac{d\rho_k}{dt}\right)_{\text{chem}} = m_k \, (k = 1, 2, 3, \ldots n)$$

plus equations describing the relaxation of the oscillatory degrees of freedom

$$\frac{\partial e_{vi}}{\partial x} v_x + \frac{\partial e_{vi}}{\partial y} v_y = \frac{e_{vie} - e_{vie}}{\tau_i} = \omega_{vi} \, (i = N_2, O_2).$$

here $\left(\frac{d\rho_k}{dt}\right)_{\text{chem}}$ is the rate of formation of the k-th component as a result of chemical reactions occurring in the mixture, τ_i is the relaxation time of the oscillatory degrees of freedom, e_{vie} is the equilibrium value of the oscillating energy.

A detailed analysis of the processes of dissociation and ionization, performed in works [4-6], allows us to analyze the air heated upon passage through a shock wave as a nine component mixture, in which the following chemical reactions occur:

$$O_2 + L \rightleftarrows 2O + L, \qquad (1)$$

$$N_2 + L \rightleftarrows 2N + L, \qquad (2)$$

$$NO + L \rightleftarrows N + O + L, \qquad (3)$$

$$O + N_2 \rightleftarrows NO + N, \qquad (4)$$

$$NO + O \rightleftarrows N + O_2, \qquad (5)$$

$$N_2 + O_2 \rightleftarrows 2NO, \qquad (6)$$

$$N + O \rightleftarrows NO^+ + e, \qquad (7)$$

$$N + N \rightleftarrows N_2^+ + e, \qquad (8)$$

$$O + O \rightleftarrows O_2^+ + e \qquad (9)$$

(L is any molecule or atom participating in a triple collision.) In addition to the chemical reactions and reactions of ionization, the excitation of oscillatory degrees of freedom of the oxygen and nitrogen molecules is taken into consideration; the internal energy of NO is assumed equilibrium. /199

All approximations related to the description of the thermodynamic properties of the components and the kinetics of the nonequilibrium processes correspond precisely with the data developed in [1], since the calculation of the supersonic portion of the blunted cone is performed using the results of this work concerning the flow around the subsonic and transonic portion. The reaction rate constants and the approximations of the equilibrium constants are taken from works [4, 6]. The chemical reaction rate constants (1)-(6) are extrapolations to higher temperatures of experimental data produced for temperatures up to 10^4°K. In calculating the dissociation rates, the varying catalytic effectiveness of the components participating in the reaction is taken into consideration. The reaction rates for ionization (7)-(9) were produced on the basis of known experimental results on the effective collision cross sections for ionization and tested by experiments up to $M \sim 27$ [4]. The dependences of relaxation times for oscillations on temperature and chemical composition are taken from work [5].

System of equations [1] for supersonic flow is hyperbolic. Its solution can be achieved using the characteristics method. The characteristics of the first and second sets are determined by the relationships

$$\frac{dy}{dx} = \frac{A\sqrt{M^2-1} \pm B}{B\sqrt{M^2-1} \mp A},$$

$$d\delta \pm \frac{1+\delta^2}{2} \cdot \frac{\sqrt{M^2-1}}{\rho w^2}\, dp \pm$$

$$\pm \frac{1+\delta^2}{4} \cdot \frac{dx}{B\sqrt{M^2-1} \mp A}\left[\frac{A}{y}j + \Phi\sqrt{A^2+B^2}\right] = 0,$$

$$A = \delta(1-\delta^2),\; B = \left(\frac{1-\delta^2}{2}\right)^2 - \delta^2,\; \delta = \mathrm{tg}\,\frac{\theta}{4}.$$

Along the line of flow, the following relationships are correct:

$$\frac{dy}{dx} = \frac{A}{B},$$

$$w\,dw + \frac{dp}{\rho} = 0,$$

$$d\rho = \frac{dp}{a^2} + \Phi\rho\, ds,$$

$$\frac{d\alpha_k}{ds} = \varphi_k = \frac{m_k}{w}, \quad k = 1, 2, \ldots n,$$ /200

$$\frac{de_{vi}}{ds} = \omega_{vi}, \quad i = N_2, O_2,$$

where

$$\Phi = \sum_{k=1}^{n}\left[\omega_{vi}\frac{a_i}{\mu_i} \cdot \frac{1}{c_pT} + \left(\frac{h_k}{c_pT} - \frac{\mu}{\mu_k}\right)\varphi_k\right],$$

$$i = N_2, O_2, N_2^+, O_2^+,$$

$$c_p = \sum_{k=1}^{n} \left(\frac{\partial h_k}{\partial T} \right)_{\substack{e_{v_{O_2}} = \text{const} \\ e_{v_{N_2}} = \text{const}}} a_k,$$

$$a^2 = \frac{\gamma p}{\rho}; \quad \gamma = \frac{1}{1 - \frac{1}{c_p \mu}},$$

c_p is the "frozen" specific heat at constant pressure.

In calculating the supersonic flow area around a blunt body, we use the boundary conditions on the body and on the shock wave in a form corresponding to an ideal gas with constant specific heat, since upon transition through the shock wave, all the physical and mechanical processes are considered frozen. Calculation of the flow field between the wall and the shock wave (see Figure 1) was performed sequentially by regions limited by the reflected characteristics: in area I, the Cauchy problem was solved, in areas II, IV, VI, etc. the flow was calculated between the characteristics of the first set and the wall, while in areas III, V, VII, etc. the flow was calculated between the characteristics of the second set and the shock wave. Characteristic CD, passing through the point of contact of the sphere with the cone, divides the entire flow area into two parts: the area of flow around the sphere (A'ABB') and the area of influence of the cone (DCEF). Characteristic CD and its corresponding characteristics reflected from the shock wave and the body are lines of discontinuity of the derivatives. The flow around the cone was sequentially calculated by regions limited by these discontinuous characteristics.

On the initial ray AB, 21 points were fixed, and 41 points were fixed on all discontinuous characteristics. The step along the wall and along the shock wave was so selected that the points along the wall and along the shock wave were located rather evenly. In order to test the correctness of the solution produced, the condition of constancy of total enthalpy and the correctness of the equation for total flow were checked. Along the wall, the total enthalpy changed by a fraction of 1%. In the flow field, the total enthalpy changed within limits of 1.5% (on the initial ray, the total enthalpy changed by 0.7%). The differences in the flow rate passing through the characteristics of the first and second sets and passing by a fixed point lay within limits of 1%. /201

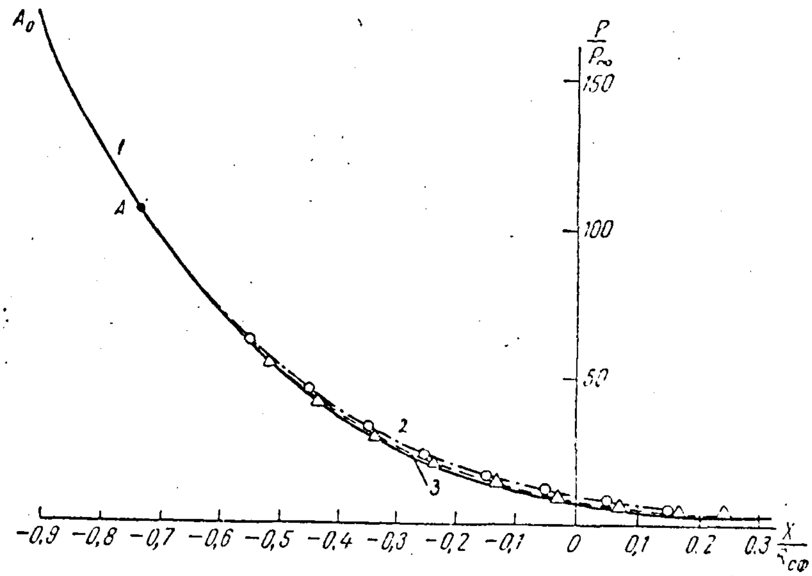

Figure 2. Distribution of Pressure Along Surface of Sphere. 1, Uneven flow; 2, Even flow; 3, Flow of ideal gas (κ = 1.4)

As an example, we calculated the flow around a blunt cone with spherical tip radius of 1.5 cm by a flow of air at $T_\infty = 250°$, $p_\infty = 10^{-3}$ atm., $M_\infty = 13$, and compared the results with equilibrium flow and flow at $\kappa = 1.4$. The results concerning the flow around the blunted cones by equilibrium and ideal flows were calculated using the method described in work [7], and place at our disposal by L. V. Pchelkina and Yu. N. D'yakonov.

Figures 2 and 3 show the distribution of pressure and density along the surface of the sphere as a function of the value of $\bar{x}$. In the subsonic and transonic areas, the calculation was performed using the method of work [1]. Beginning at point A, calculation was performed using the characteristics method. In the region of point A, the results of calculations performed using the characteristics method agreed well with the results of calculations using the method of work [1] and, by extrapolating data beyond point A using interpolation polynomials to calculate curve A_0A_1. The pressure on the surface of /202 the sphere for the nonequilibrium flow practically corresponds with the distribution of pressure for an ideal gas, the difference between the equilibrium, nonequilibrium and ideal cases being slight. The presence of dissociation in the air influences the density distribution more strongly. The density distribution for the ideal and equilibrium flows differ notably. When the nonequilibrium physical and chemical conversions are taken into consideration, the density is near its equilibrium value.

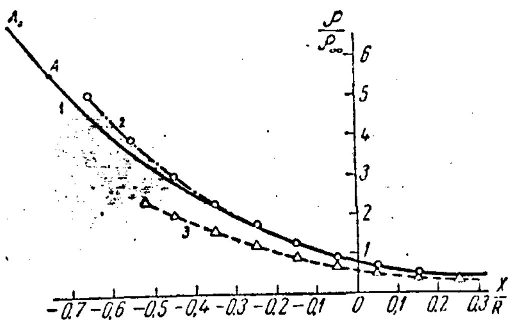

Figure 3. Distribution of Density Along Surface of Sphere. 1, Nonequilibrium flow; 2, Equilibrium flow; 3, Flow of ideal gas (κ = 1.4)

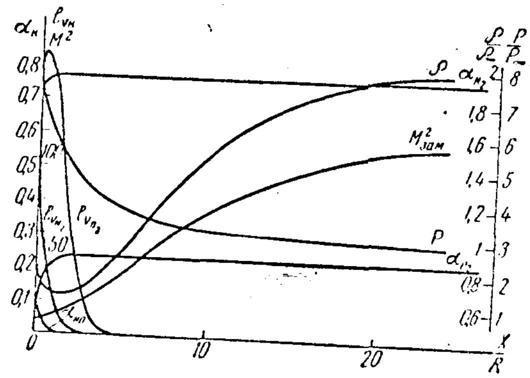

Figure 4. Distribution of Parameters Along Characteristics of First Set Passing Through Point of Contact Between Sphere and Cylinder

This calculation variant is characteristic in that most chemical reactions occur in the subsonic area. The course of these chemical reactions in the supersonic area is practically frozen. The profiles of concentrations and nonequilibrium oscillatory energies are determined by the corresponding profiles on the characteristics in the transonic area. The presence of an entropy layer leads to great transverse gradients of all parameters near the surface of the body. The gradients of concentrations and oscillatory energies are particularly great. Figure 4 shows a typical distribution of all parameters along the characteristics of the first set passing through the point of contact of the sphere with a cylinder. Downstream, the magnitude of the entropy layer decreases, the gradients of the parameters increase. Therefore, in calculating long bodies, the number of calculating points near the body should be increased.

Figures 5 and 6 show a comparison of the distributions of pressure and density along blunted cones with angles of +10°, 0°, -10° and -20° for ideal, equilibrium and nonequilibrium flow.

The nonequilibrium value of pressure in the neighborhood of the contact between the cone and the sphere is less than the ideal and equilibrium values. Downstream, the value of pressure approaches the ideal. The nonequilibrium value of density is notably higher than the equilibrium and nonequilibrium value [sic -- Tr.]. /203

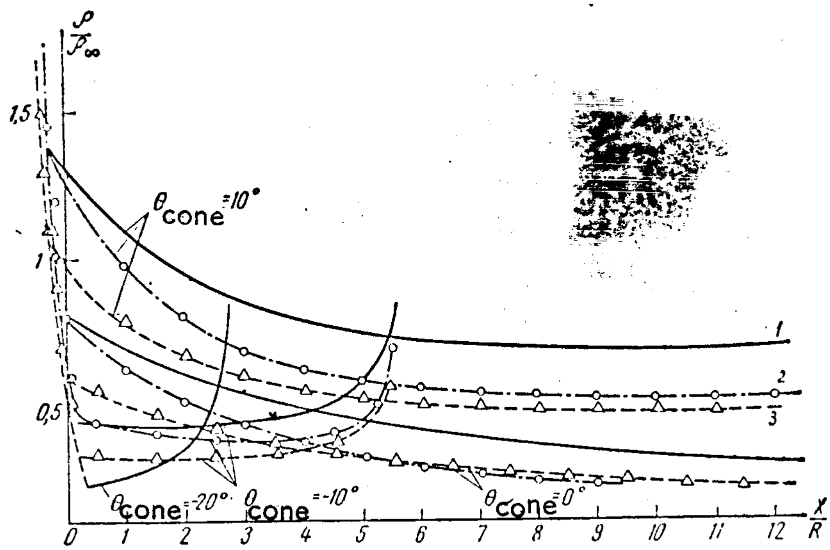

Figure 5. Distribution of Pressure Along Surfaces of Blunted Cones. 1, Nonequilibrium flow; 2, Equilibrium flow; 3, Ideal gas flow (κ = 1.4)

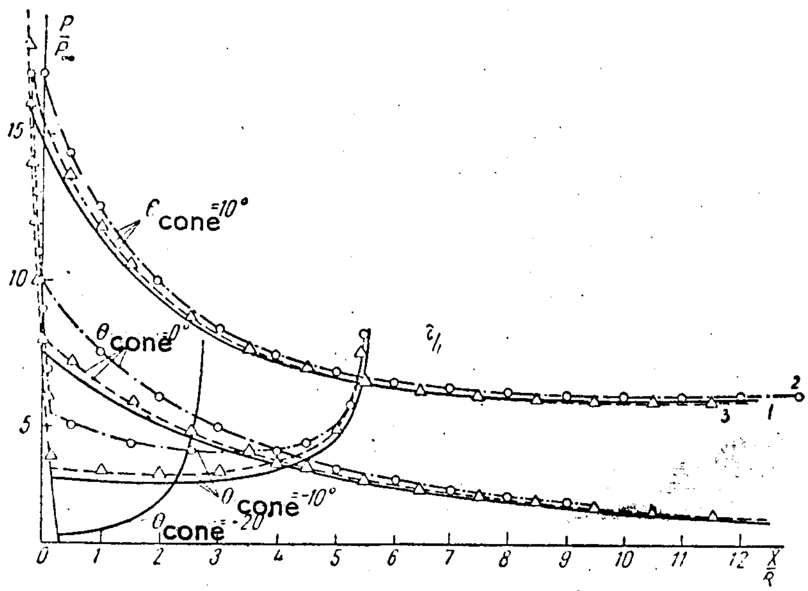

Figure 6. Distribution of Pressure Along Surfaces of Blunted Cones. 1, Nonequilibrium flow; 2, Equilibrium flow; 3, Flow of an ideal gas (κ = 1.4)

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Translated for the National Aeronautics and Space Administration under contract No. NASw-1695 by Techtran Corporation, P.O. Box 729, Glen Burnie, Maryland 21061